MA 234, Spring 2015

Assignment 9

This homework is due Monday April 6 (because of Good Friday).

There are total 60 points in this assignment. 54 points is considered 100%. If you go over 54 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 6.1–6.2 of Textbook.

(1) [10pt] Find the following integrals (either by expressing them through real and imaginary part, or by guessing the complex antiderivative):

(a) $\int_0^1 (3t+i)^2 dt$,	(c) $\int_0^2 \frac{t}{t+i} dt$,
(b) $\int_0^{\frac{\pi}{2}} \cosh(it) dt$,	(d) $\int_0^1 e^{it+2t} dt$.

(2) [5pt] Let m, n be integers. Show that

$$\int_0^{2\pi} e^{imt} e^{-int} dt = \begin{cases} 0 \text{ when } m \neq n, \\ 2\pi \text{ when } m = n. \end{cases}$$

(3) [5pt] Show that $\int_0^\infty e^{-zt} dt = \frac{1}{z}$ provided $\mathrm{Re}(z) > 0$. Why is the latter condition important?

(4) [10pt] Evaluate $\int_C x dz$ from -4 to 4 along the following contours:
(a) The polygonal path C with vertices -4, $-4+4i$, $4+4i$, and 4.
(b) The upper half of the circle $|z| = 4$.

(5) [10pt] Same question about the integral $\int_C z dz$.

(6) [10pt] By $C_r^+(a)$ we denote a circle of radius r centered at a traversed counterclockwise. By $C_r^-(a)$ we denote the same circle traversed clockwise. Evaluate the following integrals.

(a) $\int_{C_4^+(0)} z dz$.	(d) $\int_{C_2^-(0)} \frac{1}{z} dz$.	(g) $\int_{C_3^+(0)} (1/\bar{z}^2) dz$.
(b) $\int_{C_4^+(0)} \bar{z} dz$.	(e) $\int_{C_2^-(0)} (1/\bar{z}) dz$.	
(c) $\int_{C_2^+(0)} \frac{1}{z} dz$.	(f) $\int_{C_3^+(0)} \frac{1}{z^2} dz$.	

(7) [10pt]
(a) Let C be a straight line joining z_0 to z_1. Establish that $\int_C dz = z_1 - z_0$ by computing the integral explicitly (that is, by parameterizing C and plugging the parametrization in the integral).
(b) Same question for $\int_C z dz = \frac{z_1^2}{2} - \frac{z_0^2}{2}$.
(c) Let C' be a triangular contour with vertices z_0, z_1, z_2, that is a contour that consists of straight lines joining z_0 to z_1, then z_1 to z_2, and then z_2 to z_0. Show that $\int_{C'} (Az+B) dz = 0$ for any complex constants A, B. (*Hint:* $\int_{C'} (Az+B) dz = A \int_{C'} z dz + B \int_{C'} dz$.)